Skew-polynomial rings and algebraic coding theory

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Cyclic codes are one of the most studied families of block codes in classical coding theory, because they provide the algebraic framework for the construction of codes such as Reed-Solomon and BCH codes. A natural generalization of these codes are the so-called skew-cyclic codes. They are based on skew-polynomial rings in one indeterminate. The only difference from a commutative polynomial ring is that in the skew version the indeterminate does not commute with its coefficients. In this talk, we will first discuss the applications of the theory of skew-polynomial rings to algebraic coding theory. We will discuss some recent results pertaining to the distance of skew-cyclic codes in Hamming, rank and sum-rank metrics. The presentation is based on literature on skew-polynomial rings by Ore (1933) and Lam/Leroy (between 1988 and 2012), as well as literature on skew-cyclic codes by Boucher/Ulmer et al. (between 2007 and 2014), and on joint work with Lobillo, Neri and Wachter-Zeh (2021-2022).

References

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