# An Effective Version of the Grothendieck $p$-curvature Conjecture for Order One Differential Equations

Florian Fürnsinn
University of Vienna, Austria

30th Applications of Computer Algebra - ACA 2025

To a linear differential equation with polynomial coefficients over the rational numbers one can attach, for all prime numbers $p$, a linear map called the *$p$-curvature*. The Grothendieck $p$-curvature conjecture asserts that the algebraicity of a full basis of solutions of such a differential equation is equivalent to the vanishing of the $p$-curvatures for almost all prime numbers $p$. In 1974 Honda provided a proof of this conjecture for order one equations [2], by reducing the problem to a theorem of Kronecker [3], which provides a local-global criterion for the splitting of polynomials over the rational numbers. In 1985 Chudnovsky and Chudnovsky gave a new proof of Kronecker's result [1], and with it of Honda's result, using Padé approximation.

In this talk I will explain how to use the proof of the Chudnovsky brothers to make Honda's result effective. More precisely, given a linear differential equation of order one with polynomial coefficients over the rational numbers we deduce an upper bound on the number of $p$-curvatures to be computed in order to decide the algebraicity of all solutions of the equation.

This talk is based on ongoing joint work with Lucas Pannier.

## References

[1] David V. Chudnovsky and Gregory V. Chudnovsky. Applications of Padé approximations to the Grothendieck conjecture on linear differential equations. *Number theory* (New York, 1983–84). Vol. 1135. Lecture Notes in Math. Springer, Berlin, 1985, pp. 52–100.

[2] Taira Honda. Algebraic differential equations. *Symposia Mathematic.*, Vol. XXIV (Sympos., INDAM, Rome, 1979). London-New York: Academic Press, 1981, pp. 169–204.

[3] Leopold Kronecker. Über die Irreducibilität von Gleichungen *Monatsberichte der Königlichen Preussische Akademie des Wissenschaften zu Berlin* (1880), pp. 155–163.