Solving bihomogeneous polynomial systems with a zero-dimensional projection

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In this talk, we study bihomogeneous systems defining, non-zero dimensional, biprojective varieties for which the projection onto the first group of variables results in a finite set of points. To compute (with) the 0-dimensional projection and the corresponding quotient ring, we introduce linear maps that greatly extend the classical multiplication maps for zero-dimensional systems, but are not those associated to the elimination ideal; we also call them multiplication maps. We construct them using linear algebra on the restriction of the ideal to a carefully chosen bidegree or, if available, from an arbitrary Gröbner bases. The multiplication maps allow us to compute the elimination ideal of the projection, by generalizing FGLM algorithm to bihomogenous, non-zero dimensional, varieties. We also study their properties, like their minimal polynomials and the multiplicities of their eigenvalues, and show that we can use the eigenvalues to compute numerical approximations of the zero-dimensional projection. Finally, we establish a single exponential complexity bound for computing multiplication maps and Gröbner bases, that we express in terms of the bidegrees of the generators of the corresponding bihomogeneous ideal. This talk is based on joint work with Laurent Busé, Carles Checa and Elias Tsigaridas.