Invariant Polynomials and Cyclic Line Spreads

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Flag-transitive linear spaces have been fully classified for all but a small class of possible automorphism groups. For one of the remaining open cases, which arise from line-spreads fixed by a cyclic group, Bamberg and Pauley showed an equivalent characterisation in terms of polynomials $P(x) \in \mathbb{F}_{q^2}$ over a finite field with certain properties. The requirements on $P(x)$ are very similar, though not identical, to requirements for a related polynomial to define a permutation.

In [1] we fully solved the case of cubic polynomials using this approach. Key to the method involved the factorisation of a related two-variable polynomial $H_P(z, w)$, or plane curve. In this work we study the case of polynomials of arbitrary degree which define a flag-transitive linear space. We focus on the case where the aforementioned $H_P(z, w)$ splits into factors of low degree.

We show that this requires that $P(x)$ is invariant under certain elements $[\Psi]$ of $\mathrm{PGL}(2, \overline{\mathbb{F}_q})$. Polynomials of this form have been well-studied, from Stichtenoth-Topuzoglu, through Brochero Martinez-Garefalakis-Reis-Tzanaki, and beyond. In [2], [3] we prove further necessary conditions on $[\Psi]$ and $P(x)$, construct new families generalising those of Feng-Lu, and classify some small degrees. We will present some remaining open problems.

References

[1] Cian Jameson and John Sheekey. Cyclic 2-Spreads in $V(6, q)$ and Flag-Transitive Affine Linear Spaces, *Finite Fields and their Applications*, 98:102463, 2024.

[2] Cian Jameson. Classifying Flag-Transitive Linear Spaces using Cyclic Line-Spreads and Orbit Polynomials, *PhD Thesis*.

[3] Cian Jameson and John Sheekey. Orbit Polynomials and Cyclic Line Spreads, *in preparation*.