Braid monodromy computations using certified path tracking

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Let $f \in \mathbb{C}[t, x]$. The set of roots of f in x when t moves continuously along a loop in $\mathbb{C}$ defines a braid, provided that t avoids a certain set of critical values. Artin proved that braids can be described in terms of elementary generators, and our goal is to compute such a decomposition for the braid induced by the displacement of the roots of f. Starting with the algebraic input f, we first numerically track its roots using certified homotopy continuation. This procedure outputs disjoint tubular neighborhoods each containing a strand of the induced braid. Although this output is by nature numerical, we can recover the braid expressed in terms of Artin's generators from it. This discrete description is certified, even though the intermediate step involves numerical computations. We provide a Rust implementation of the second step that can be piped with Algpath, a certified path tracking software, allowing for certified braid monodromy computations.