On the minimum weight of some geometric codes

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Assume p is a prime and m, h are two positive integers. Let $\Sigma = \mathrm{PG}(m, q)$ be the m-dimensional projective space over the Galois field $\mathbb{F}_q$ where $q = p^h$, and denote by the symbol $\mathcal{D}_\Sigma(m, q)$ the $2-(v, q+1, 1)$ design of points and lines of Σ; hence, with $v = \frac{q^{m+1}-1}{q-1}$. The p-ary code $\mathcal{C} = \mathcal{C}_\Sigma(m, q)$ associated with such a design is the $\mathbb{F}_p$-subspace generated by the incidence vectors of the blocks of the corresponding design. Also, the dual $\mathcal{C}^\perp$ of $\mathcal{C}$ is the $\mathbb{F}_p$-subspace of vectors of $\mathbb{F}_q^v$ which are orthogonal to all vectors of $\mathcal{C}$ (under the standard inner product). These are particular examples of so called *geometric codes*.

Unlike for codes derived from the designs of points and subspaces of Σ, the situation regarding the minimum weight of geometric codes is not as clear, and therefore its study is more challenging. In [3] the authors reduced this problem to the above mentioned case of points and lines of a projective space of suitable dimension. In [1] Bagchi and Inamdar proven that the minimum weight of $\mathcal{C}_\Sigma^\perp(m, q)$ is bounded from below by the value $2\left(\frac{q^m-1}{q-1}\left(1-\frac{1}{p}\right)+\frac{1}{p}\right)$.

This type of problem in coding theory can be quite naturally translated into one concerning with the cardinality of sets or *multi-sets* of points in projective or affine space with special intersection properties with respect to certain subspaces, as shown for instance in [2]. Using this geometrical approach and exploiting properties of certain kind of polynomial, in this talk, we will show a significant improvement of the bound stated in 2002 by Bagchi and Inamdar, in the case when $h > 1$, and $m, p > 2$.

References

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