Towards the classification of scattered binomials

Francesco Ghiandoni
University of Perugia, Italy

30th Applications of Computer Algebra - ACA 2025

Joint work with Daniele Bartoli, Alessandro Giannoni and Giuseppe Marino.

Let f be an $\mathbb{F}_q$-linear function over $\mathbb{F}_{q^n}$. If $U = \{(x, f(x)) : x \in \mathbb{F}_{q^n}\}$ defines a maximum scattered $\mathbb{F}_q$-subspace of $\mathbb{F}_{q^n} \times \mathbb{F}_{q^n}$, f is said to be a scattered polynomial. So far, very few examples of such polynomials are known for each value of n and q. In particular, the only known families of scattered binomials are

(LP) $f(x) = \delta x^{q^s} + x^{q^{n-s}}$, with $\gcd(s, n) = 1$ and $\delta^{(q^n-1)/(q-1)} \neq 1$;

(CMPZ) $f(x) = \delta x^{q^s} + x^{q^{n/2+s}}$, for $n = 6, 8$ and certain choices of δ.

In this talk, we will show that, at least when n is a prime integer, scattered binomials are of LP type only. Finally, a classification of scattered binomials over $\mathbb{F}_{q^n}$ for $n \leq 8$ is exhibited.

References

[1] D. Bartoli, B. Csajbók, M. Montanucci. On a conjecture about maximum scattered subspaces of $\mathbb{F}_{q^6} \times \mathbb{F}_{q^6}$, *Linear Algebra Appl.*, **631**:111–135, 2021.

[2] B. Csajbók, G. Marino, O. Polverino, C. Zanella. A new family of MRD-codes, *Linear Algebra Appl.*, **548**:203–220, 2018.

[3] G. Lunardon, R. Trombetti, Y. Zhou. Generalized twisted Gabidulin codes, *J. Combin. Theory Ser. A*, **159**:79–106, 2018.

[4] J. Sheekey. A new family of linear maximum rank distance codes, *Adv. Math. Commun.*, **10**(3):475–488, 2016.

[5] M. Timpanella, G. Zini. On a family of linear MRD codes with parameters $[8 \times 8, 16, 7]_q$. *Des. Codes Cryptogr.* , **92**(3):507–530, 2024.