A generalization of Magnus duality

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Let X be the finite graded set $X = B + Z$ (where $B = \{b_1, \ldots, b_M\}$ and $Z = \{z_1, \ldots, z_N\}$) and $\mathbb{K}$ being a fixed ring. In this talk, we first review a Zinbiel bialgebra structure over the associative algebra $\mathbb{K}\langle X\rangle$ and its graded dual studied in [1], [2] and [5]. We then use the concept of the classical Lazard elimination to construct a $\mathbb{K}$-linear basis of $\mathbb{K}\langle X\rangle$ which is called Magnus basis [3]. As the main purpose, we will explain how to use these generalized bialgebra structures over $\mathbb{K}\langle X\rangle$ to provide combinatorial tools in order to obtain the duality of Magnus basis. We claim that the duality can be automatically approached to any graded set $X = B + Z$ where $B = \{b_\gamma\}_{\gamma\in\Gamma}$ and $Z = \{z_\lambda\}_{\lambda\in\Lambda}$ (Γ, Λ are nonempty index sets). In case $X = B + Z$ where $B = \{x_0\}$ and $Z = \{x_\lambda\}_{\lambda\in\Lambda}$ (Λ: a nonempty index set, for example $\mathbb{N}_+$), the Magnus duality was appeared in [4], Theorem 3.2 to derive a formula of Le-Murakami, Furusho type that expresses arbitrary coefficients of a group-like series $\mathcal{J} \in \mathbb{K}\langle\langle x_0, x_1\rangle\rangle$ in terms of the "regular" coefficients of $\mathcal{J}$ ([4], Theorem 4.1).

This is based on join works with Prof. Gerard Duchamp and Prof. Vincel Hoang Ngoc Minh [6].

References

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