Multiplicative structure of some multivariate functions

Jean-Yves Enjalbert
Lycée Jean-Batiste Corort, France

30th Applications of Computer Algebra - ACA 2025

We will examine various families of multivariate functions, including the well-known multivariate functions.

$$\zeta(s_1;\ldots;s_n)=\sum_{n_1>\ldots>n_r>0}\frac{1}{n_1^{s_1}\ldots n_r^{s_r}}$$

For each family, we propose an efficient coding on an alphabet X, and transfer the multiplicative laws of their algebras to the alphabet X using the ϕ-stuffle $\sqcup\!\sqcup_\phi$, defined recursively by:

$$\forall(a,b)\in X^2,\forall(u,v)\in\left(X^*\right)^2,\quad au\sqcup\!\sqcup_\phi vb=a(u\sqcup\!\sqcup_\phi bv)+b(au\sqcup\!\sqcup_\phi v)+\phi(a,b)(u\sqcup\!\sqcup_\phi v)\ ,$$

The following will be studied:

- the explanation of ϕ according to various contexts.
- the conditions on ϕ to benefit from Radford's theorem, i.e., having a transcendence basis on the algebra generated by the family considered. Note that the basis is given explicitly, with a construction method allowing an efficient implementation of the algebra.
- the conditions on ϕ to determine the possibility of dualizing the mixing law, and therefore of a Hopf algebra structure.

The study will conclude by returning to the families introduced in the introduction.