

On Kashiwara-Vergne Lie algebra and double shuffle Lie algebra in mould theory

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In 2012, Schneps ([6]) showed that there exists an embedding between the double shuffle Lie algebra introduced by Racinet ([4]) and the Kashiwara-Vergne Lie algebra introduced by Alekseev and Torossian ([1]). In the proof of this embedding, a relation called the *senary relation* is used, which is a notion introduced in Ecalle's study ([2]) of multiple zeta values using mould theory. On Lie algebras and embeddings above, bigraded or elliptic versions have also been studied ([3], [5]). In my talk, I will explain the above topics and recent topic ([7]) as much as time permits.

References

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