Iterated strongly normal extensions and nonlinear differential equations

V. Ravi Srinivasan and Partha Kumbhakar
IISER Mohali, India

30th Applications of Computer Algebra - ACA 2025

In this talk, I will sketch various results describing the structure of differential subfields of iterated strongly normal extensions. These results will then be used to study differential algebraic dependence of generic solutions in iterated strongly normal extensions of nonlinear differential equations. Our results along with the work [1], immediately proves that the Lotka-Volterra system

$$\begin{cases} y_1' = \alpha y_1 + \beta y_1 y_2 \\ y_2' = \gamma y_2 + \delta y_1 y_2 \end{cases}$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{C} \setminus 0$, has a generic solution in an iterated strongly normal extension E of $\mathbb{C}$ if and only if $\alpha = \gamma$, which in turn holds if and only if the generic solution is in an elementary extension of $\mathbb{C}$ and that the Poizat differential equation $y'' = y' f(y)$, where $f \in \mathbb{C}(y)$, has no generic solution in an iterated strongly normal extension of $\mathbb{C}$.

References

[1] Christine Eagles and Léo Jimenex. Internality of autonomous algebraic differential equations. *arXiv*:2409.01863v1.