

A Reduce package for Differential Operators in Mathematical Physics and Theoretical Physics

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30th Applications of Computer Algebra - ACA 2025

The mathematical subject ‘Geometry of Differential Equations’, although not mainstream, covers many topics which have significant overlap with several other branches of Mathematics, like symmetries and conservation laws of ODEs and PDEs, Hamiltonian or symplectic formalism for ODEs and PDEs, integrability. See the books [1] and [5] for an overview. The CDE package was developed in Reduce with the purpose of providing a tool for computations in the above field [4]. This package has already been used in a number of papers (e.g. [2], [3] and [6] to [8]).

Recent developments of the capabilities of CDE go in the direction of computing with differential operators. We will show how newly added software can be used for typical computations related with the search of Lax pairs (joint work with R. La Scala, Un. of Bari, Italy), or can provide an environment for calculations in Supersymmetric Quantum Mechanics (joint work with L. Miranda and F. Toppan CBPF – Rio De Janeiro, Brazil).

References

- [1] A. V. Bocharov, V. N. Chetverikov, S. V. Duzhin, N. G. Khor’kova, I. S. Krasil’shchik, A. V. Samokhin, Yu. N. Torkhov, A. M. Verbovetsky, and A. M. Vinogradov. *Symmetries and Conservation Laws for Differential Equations of Mathematical Physics*. Monograph. Amer. Math. Soc., 1999. I. S. Krasil’shchik and A. M. Vinogradov editors.
- [2] M. Casati, P. Lorenzoni, D. Valeri, and R. Vitolo. Weakly nonlocal Poisson brackets: tools, examples, computations. *Computer Physics Communications*, 274:108284, 2022, DOI: <https://doi.org/10.1016/j.cpc.2022.108284>, arXiv: <https://arxiv.org/abs/2101.06467>.
- [3] M. Casati, P. Lorenzoni, and R. Vitolo. Three computational approaches to weakly nonlocal poisson brackets. *Stud. Appl. Math.*, 144(4):412–448, 2020, DOI: 10.1111/sapm.12302, arXiv: <https://arxiv.org/abs/1903.08204>.
- [4] J. Krasil’shchik, A. Verbovetsky, and R. Vitolo. *The symbolic computation of integrability structures for partial differential equations*. Texts and Monographs in Symbolic Computation. Springer, 2018. ISBN 978-3-319-71654-1; see http://gdeq.org/Symbolic_Book for downloading program files that are discussed in the book.
- [5] P.J. Olver. *Applications of Lie Groups to Differential Equations*. Springer-Verlag, 2nd edition, 1993.
- [6] S. Opanasenko and R. Vitolo. Bi-Hamiltonian structures of wdv type. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.*, 480:20240249, 2024, DOI: <https://doi.org/10.1098/rspa.2024.0249> arXiv: <https://arxiv.org/abs/2407.17189>.
- [7] J. Vařicek and R. Vitolo. WDVV equations and invariant bi-Hamiltonian formalism. *J. of High Energy Physics*, 129, 2021, arXiv: <https://arxiv.org/abs/2104.13206>. Calculations in GitHub: <https://github.com/Jakub-Vasicek/WDVV-computations>.
- [8] R. Vitolo. Computing with Hamiltonian operators. *Comput. Phys. Comm.*, 244:228–245, 2019, arXiv: <https://arxiv.org/abs/1808.03902>.