

Affirmative answer to the Question of Leroy and Matczuk on injectivity of endomorphisms of semiprime left Noetherian rings with large images

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The class of semiprime left Goldie rings is a huge class of rings that contains many large subclasses of rings – semiprime left Noetherian rings, semiprime rings with Krull dimension, rings of differential operators on affine algebraic varieties and universal enveloping algebras of finite dimensional Lie algebras to name a few. In 2013, the paper, ‘Ring endomorphisms with large images,’ *Glasg. Math. J.* 55 (2013), no. 2, 381–390, A. Leroy and J. Matczuk posed the following question [1]:

If a ring endomorphism of a semiprime left Noetherian ring has a large image, must it be injective?

The aim of the paper is to give an affirmative answer to the Question of Leroy and Matczuk and to prove the following more general results.

Theorem. (Dichotomy) *Each endomorphism of a semiprime left Goldie ring with large image is either a monomorphism or otherwise its kernel contains a regular element of the ring ($\Leftrightarrow$ its kernel is an essential left ideal of the ring). In general, both cases are non-empty.*

Theorem. *Every endomorphism with large image of a semiprime ring with Krull dimension is a monomorphism.*

Theorem. (Affirmative answer to the Question of Leroy and Matczuk) *Every endomorphism with large image of a semiprime left Noetherian ring is a monomorphism.*

References

- [1] A. Leroy and J. Matczuk Ring endomorphisms with large images *Glasg. Math. J.*, 55, no. 2: 381–390, 2006.