On the maximal spread of symmetric Bohemian matrices

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Let A be a square matrix with entries in $\mathbb{R}$. The spread of A is defined as the maximum of the distances between the eigenvalues of A. Let $S_m[a, b]$ denote the set of all $m \times m$ symmetric matrices with entries in the real interval $[a, b]$ and let $S_m\{a, b\}$ be the subset of $S_m[a, b]$ of Bohemian matrices with population from only the extremal elements $\{a, b\}$. S. M. Fallat and J. J. Xing in 2012 proposed the following conjecture: the maximum spread in $S_m[a, b]$ is attained by a rank 2 matrix in $S_m\{a, b\}$. X. Zhan had proved previously that the conjecture was true for $S_m[-a, a]$ with $a > 0$. We will show how to interpret this problem geometrically, via polynomial resultants, in order to be able to treat this conjecture from a computational point of view. This will allow us to prove that this conjecture is true for several formerly open cases.

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