From Smith forms to spectra to iterative algorithms for sparse integer matrices

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Integer matrices are often characterized by the lattice of combinations of their rows or columns. This is captured nicely by the Smith canonical form, a diagonal matrix of invariant factors, to which any integer matrix can be transformed through left and right multiplication by unimodular matrices. Algorithms for computing Smith forms have seen dramatic improvements over the past 40 years, but effective algorithms for large sparse matrices still need improvement.

Integer matrices also possess complex eigenvalues and eigenvectors, and every such matrix is similar to a unique one in Jordan canonical form. There is a wealth of numerical methods for computing eigenvalues, and Krylov-type algorithms are effective for sparse matrices.

It would seem a priori that the invariant factors and the eigenvalues would have little to do with each other. Yet we will show that for "almost all" matrices the invariant factors and the eigenvalues are equivalent under a p-adic valuation, in a very precisely counted sense.

A much-hoped-for link is then explored for fast computation of Smith forms of sparse integer matrices, via the better understood algorithms for computing eigenvalues and effective preconditioning.

This is joint work with Mustafa Elsheikh.