

Tools for fast computation of integer matrix normal forms

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In this talk we describe a number of tools which are helpful for creating fast algorithms for matrix normal forms of integer matrices. These tools include minimal denominators, Smith massagers, integer relation bases and partial linearization of integer matrices. This talk should be viewed as an introduction to the later talk by A. Storjohann.

We describe a number of tools used either explicitly or implicitly in a series of papers [1] to [3] for fast computation of Hermite and Smith normal forms of integer matrices. The primary tool used is a Smith Massager, a pair (S, F) which for a given nonsingular A allows us to approximate A^{-1} by a rational expression $F \cdot S^{-1}$ with S diagonal.

References

- [1] S. Birmpilis, G. Labahn, and A. Storjohann. A Las Vegas algorithm for computing the Smith form of a nonsingular integer matrix. In *Proc. Int'l. Symp. on Symbolic and Algebraic Computation: ISSAC'20*, page 38–45, New York, NY, USA, 2020. ACM.
- [2] S. Birmpilis, G. Labahn, and A. Storjohann. A fast algorithm for computing the Smith normal form with multipliers for a nonsingular integer matrix. *Journal of Symbolic Computation*, 116: 146–182, 2023.
- [3] Birmpilis, S. and Labahn, G. and Storjohann, A., A cubic algorithm for computing the Hermite normal form of a nonsingular integer matrix, *ACM Transactions of Algorithms*, 19: 1–36, 2023.