

Integro-differential rings and generalized shuffle relations

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In this talk, we discuss the fundamental theorem of calculus and its algebraic implications in differential rings, allowing for functions with singularities and a generalized notion of evaluation. We give an overview of integro-differential rings and present several examples. This approach generalizes results such as shuffle relations for nested integrals and the Taylor formula, incorporating additional terms to account for singularities [1].

In general, not every element of a differential ring has an antiderivative in the same ring. Starting from a commutative differential ring and a direct decomposition into integrable and non-integrable elements, we outline aspects of the construction of the free integro-differential ring [2]. This integro-differential closure contains, in particular, all nested integrals over elements of the original differential ring.

References

- [1] C.G. Raab and G. Regensburger. The fundamental theorem of calculus in differential rings. *Adv. Math.*, 447:109676, 2024.
- [2] C.G. Raab and G. Regensburger. The integro-differential closure of a commutative differential ring. In preparation, 2025.