

A computer-aided construction of non-homeomorphic double Kodaira fibrations that possess the same biregular invariants

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Let Σ_b be a closed Riemann surface of genus b . We investigate finite quotients G of the pure braid group on two strands $P_2(\Sigma_b)$ that do not factor through $\pi_1(\Sigma_b \times \Sigma_b)$. Building on previous work on special systems of generators on finite groups called *diagonal double Kodaira structures*, we prove that if G has not order 32, then $|G| \geq 64$. We completely classify the cases where equality holds (see [8]). As a geometric application of these algebraic results, we construct two 3-dimensional families of double Kodaira fibrations with the same biregular invariants and Betti numbers but different fundamental groups. When investigating groups of order 64, the computational algebra system GAP4 is central to our approach. Code is available on GitHub, [5]. This is a joint work with Francesco Polizzi.

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