

Combinatorics of Schubert Cells in Random Network Coding

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In 2009 Etzion and Silberstein provided a combinatorial upper bound on the largest dimension of a space of matrices over a finite field whose nonzero matrices are supported on a given Ferrers diagram and all have rank lower bounded by a fixed positive integer r . In the same paper, they also conjectured that such an upper bound is always tight. Since then, their conjecture has been verified in a number of cases, but as of today it still remains widely open. In this work, we investigate the notion of reducibility of Ferrers diagrams: a diagram $\mathcal{D}$ reduces to $\mathcal{D}'$ if an optimal matrix space supported on $\mathcal{D}$ can be obtained by shortening and/or inclusion of an optimal matrix space supported on $\mathcal{D}'$. This induces a natural notion of irreducibility of Ferrers diagrams, and the validity of the conjecture for irreducible diagrams implies the validity of the full conjecture. Moreover, following this notion, we can provide the Hasse diagram of Young's lattice with an orientation. This produces a directed graph in which sources correspond to irreducible diagrams. This is a Joint work with Hugo Sauerbier Couvée.