Gröbner bases native to finitely generated commutative algebras with term order, with application to the Hodge algebra of minors

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Standard Gröbner basis methods are often too inefficient to handle even small cases arising in areas such as computational complexity theory—for instance, the orbit closure of the 3×3 determinant in geometric complexity theory. Motivated by this, we develop a theory of Gröbner bases tailored to algebras with straightening law (ASLs, or Hodge algebras), and more generally to any finitely generated commutative algebra over a field $\mathbb{F}$ admitting a suitable term order. We instantiate this theory in the bideterminant ASL on a polynomial ring—generated by products of minors of a variable matrix—defining what we call bd-Gröbner bases. This framework packages several results on bideterminants in a clean form, including a one-line proof of a bd-Gröbner basis for the ideal of t-minors for any t, which is universal in our sense. While ordinary Gröbner bases for such ideals are known, their proofs are more involved.